IV International Workshop on Modern Trends in Geomechanics
G. Viggiani, D. Salciarini, F. Silvestri, C. Tamagnini, G.M.B. Viggiani
(Eds) Assisi, May 16–18, 2016

VISCOUS REGULARIZATION FOR CAM-CLAY PLASTICITY: HOW TO HANDLE SUBCRITICAL SOFTENING

Riccardo Conti[1], Antonio DeSimone[2], Claudio Tamagnini[‡]

[1]Università degli Studi di Roma Niccolò Cusano, Italy
e-mail: riccardo.conti@unicusano.it

[2]SISSA-International School for Advanced Studies, Italy
e-mail: desimone@sissa.it

[3]Dipartimento di Ingegneria Civile e Ambientale
Università degli Studi di Perugia, Italy
e-mail: claudio.tamagnini@unipg.it

Keywords: Cam-Clay, elasto-plasticity, subcritical softening, visco-plasticity.

Summary. Cam-Clay models are considered as the paradigmatic example of hardening plasticity models exhibiting pressure dependence and dilation- related hardening/softening. Depending on the amount of softening exhibited by the material, the equations governing the elastoplastic evolution problem may become ill-posed, leading to either no solutions or two solution branches (critical and sub-critical softening). In this work, an algorithm for the numerical integration of the Cam-Clay model with adaptive viscoplastic regularization is presented, allowing the numerical treatment of stressstrain jumps in the constitutive response of the material. Applications of the algorithm to standard compression tests are discussed.

1 ABSTRACT

Within the framework of continuum mechanics, the mechanical behaviour of geomaterials is often described through a rate-independent elastoplastic theory. In this field, the Cam-Clay model was firstly proposed to describe, from a phenomenological point of view, the observed behaviour of fine-grained soils. In this context, the yield function $f(\sigma, p_c)$ is defined in terms of the current stress tensor σ and the preconsolidation pressure p_c. During a plastic process, the yield surface may either expand (hardening behaviour), or shrink (softening behaviour), or remain unaltered (perfectly plastic behaviour at 'critical' state), depending on the sign of the hardening modulus $H(\sigma, p_c) = -\frac{\partial f}{\partial p_c} \cdot \dot{p}_c$ (see *e.g.* [2]).

In rate-independent elastoplasticity, an incremental loading is admissible if satisfies the classical Kuhn-Tucker complementary conditions ($\gamma \geq 0\,, f \leq 0\,, \gamma f = 0$) and the consistency requirement ($\gamma \dot{f} = 0$), where γ is the plastic multiplier. During a strain-driven process, γ is related to the current strain rate by (see *e.g.* [2]):

$$\gamma = \frac{1}{H - H_c} \frac{\partial f}{\partial \sigma} \cdot \mathbb{C}\dot{\varepsilon} \tag{1}$$

where $\mathbb{C}$ is the fourth-order elastic stiffness tensor and $H_c = -\frac{\partial f}{\partial \sigma} \cdot \mathbb{C} \frac{\partial f}{\partial \sigma}$ is the so-called critical hardening modulus for an associated flow rule. It is well known that, if $H - H_c > 0$, then the elastoplastic evolution problem has a unique solution under pure strain control conditions. However, for admissible stress states that do not satisfy the inequality (*i.e.* for critical and sub-critical softening conditions), it follows from Eq. (1) that the evolution equations are not well posed.

Recently, a method was proposed to handle subcritical softening in Cam-Clay plasticity (see *e.g.* [1]). The heuristic idea is that, by adopting a viscoplastic regularization for the equations of the rate-independent evolution problem, a unique solution can be found even in critical and sub-critical softening conditions. Here, sudden jumps (time discontinuities) of both the stress state and the internal variable will occur. They can be computed by adopting a rescaled (fast) time $s := \frac{1}{\tau}t$, where τ is the viscosity parameter (see Fig. 1 for a schematic representation).

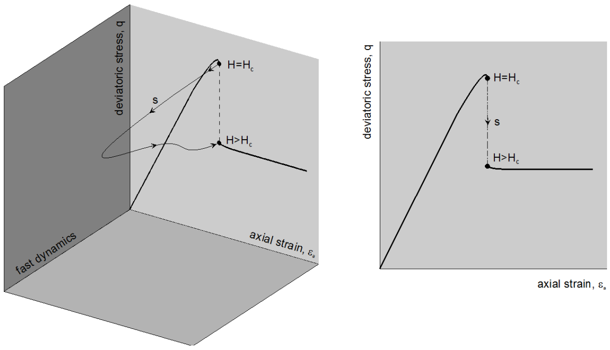
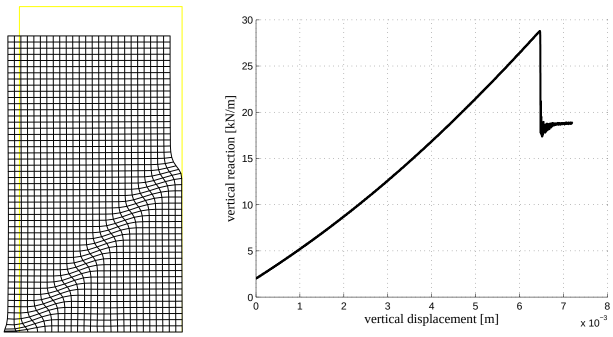


Figure 1: Stress-strain behaviour in Cam-Clay plasticity before, during and after critical softening

In this work, an algorithm for the numerical integration of the equations which govern the regularized viscoplastic Cam-Clay model is presented. The algorithm belongs to the class of fully implicit return mapping schemes [2], slightly rearranged to take into account the viscous nature of inelastic deformations. By exploiting the mathematical properties of the governing equations, the same structure can be used also for the integration during the jump phases. Numerical examples of standard compression tests, carried out both as a single element test and as a boundary value problem (Fig. 2), are presented in order to demonstrate the ability of the proposed regularization technique to handle both localization phenomena (spatial discontinuities) and critical softening (time discontinuities).



(a) (b)

Figure 2: Plane strain compression test with fast dynamics activation: (a) deformed mesh (displacement magnification = 1.0) and (b) Vertical reaction ws vertical displacement diagram.

REFERENCES

[1] G. Dal Maso, A. DeSimone, and F. Solombrino. Quasistatic evolution for cam-clay plasticity: a weak formulation via viscoplastic regularization and time rescaling. *Calc. Var.*, 40:125–181, 2011.

[2] J. Simo and T. Hughes. *Computational Inelasticity*. Springer, NY, 1997.